Digital Image Processing

Lecture 1
Introduction & Fundamentals

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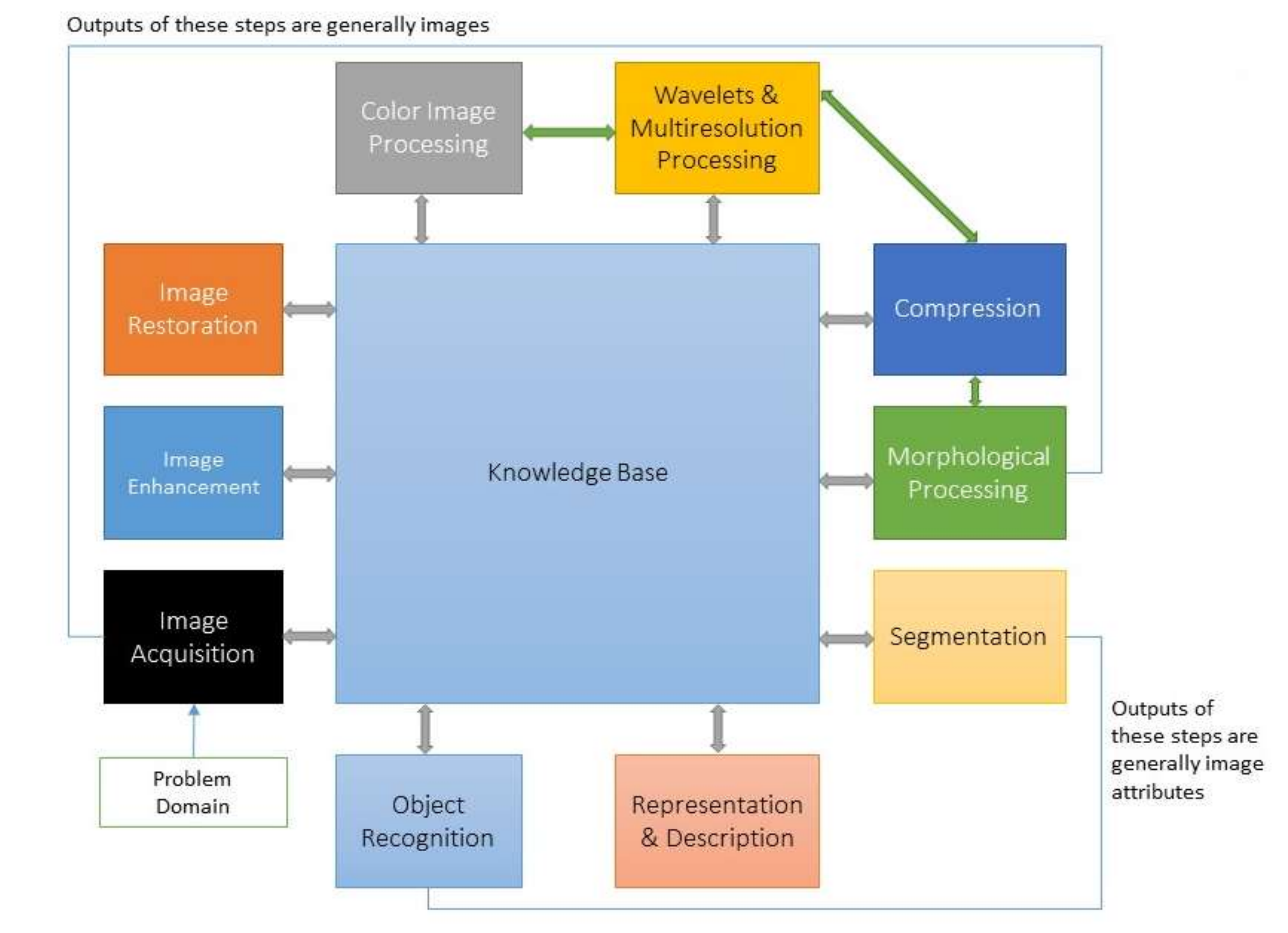
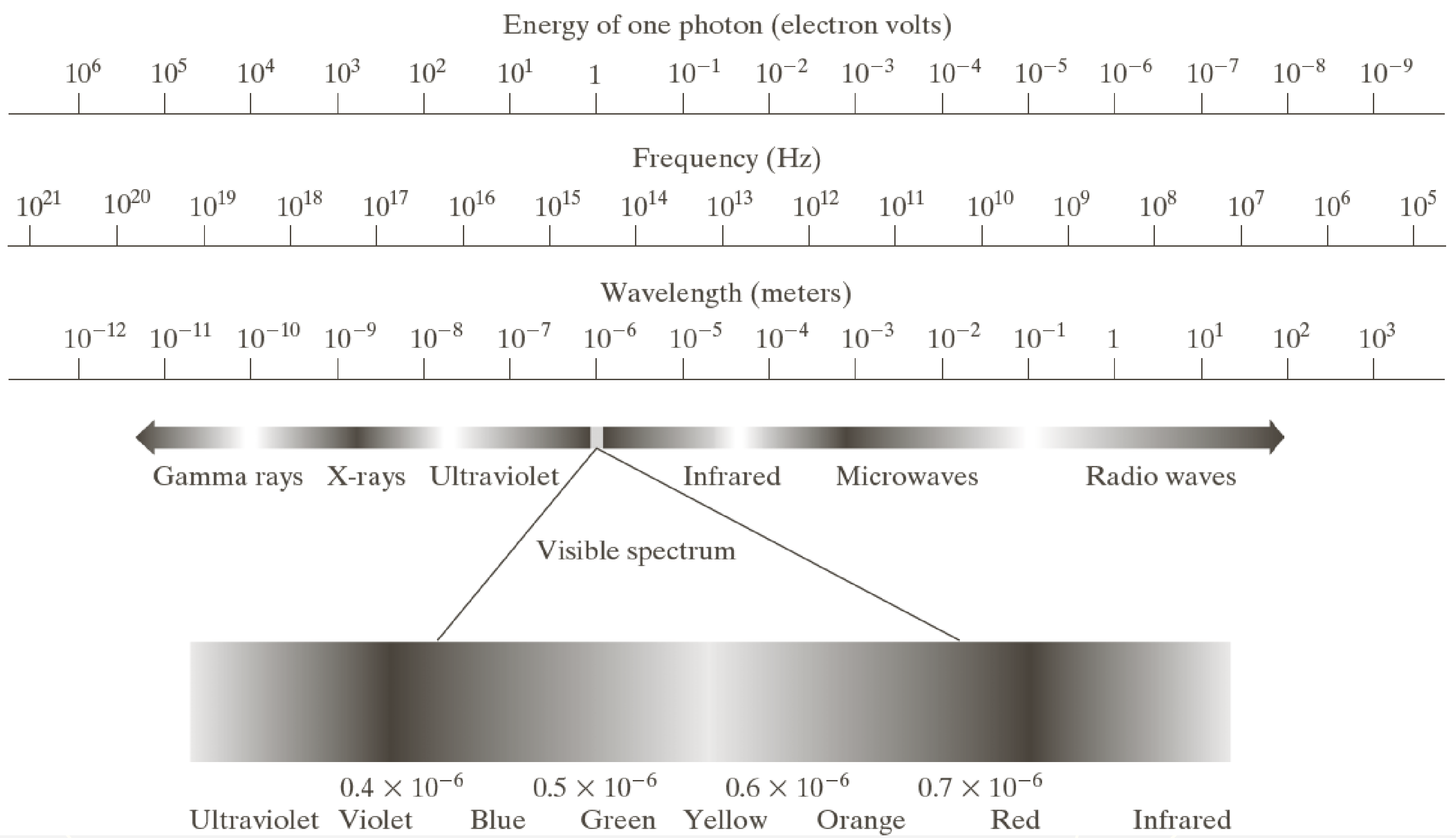
Fundamental Steps in DIP



Light and EM Spectrum



$$c = \lambda \nu \qquad E = h\nu, \quad h: \text{ Planck's constant.}$$

Light and EM Spectrum

- The colors that humans perceive in an object are determined by the nature of the light reflected from the object.

e.g. green objects reflect light with wavelengths primarily in the 500 to 570 nm range while absorbing most of the energy at other wavelength

Light and EM Spectrum

- Monochromatic light: void of color

  Intensity is the only attribute, from black to white

  Monochromatic images are referred to as **gray-scale** images

- Chromatic light bands: 0.43 to 0.79 um

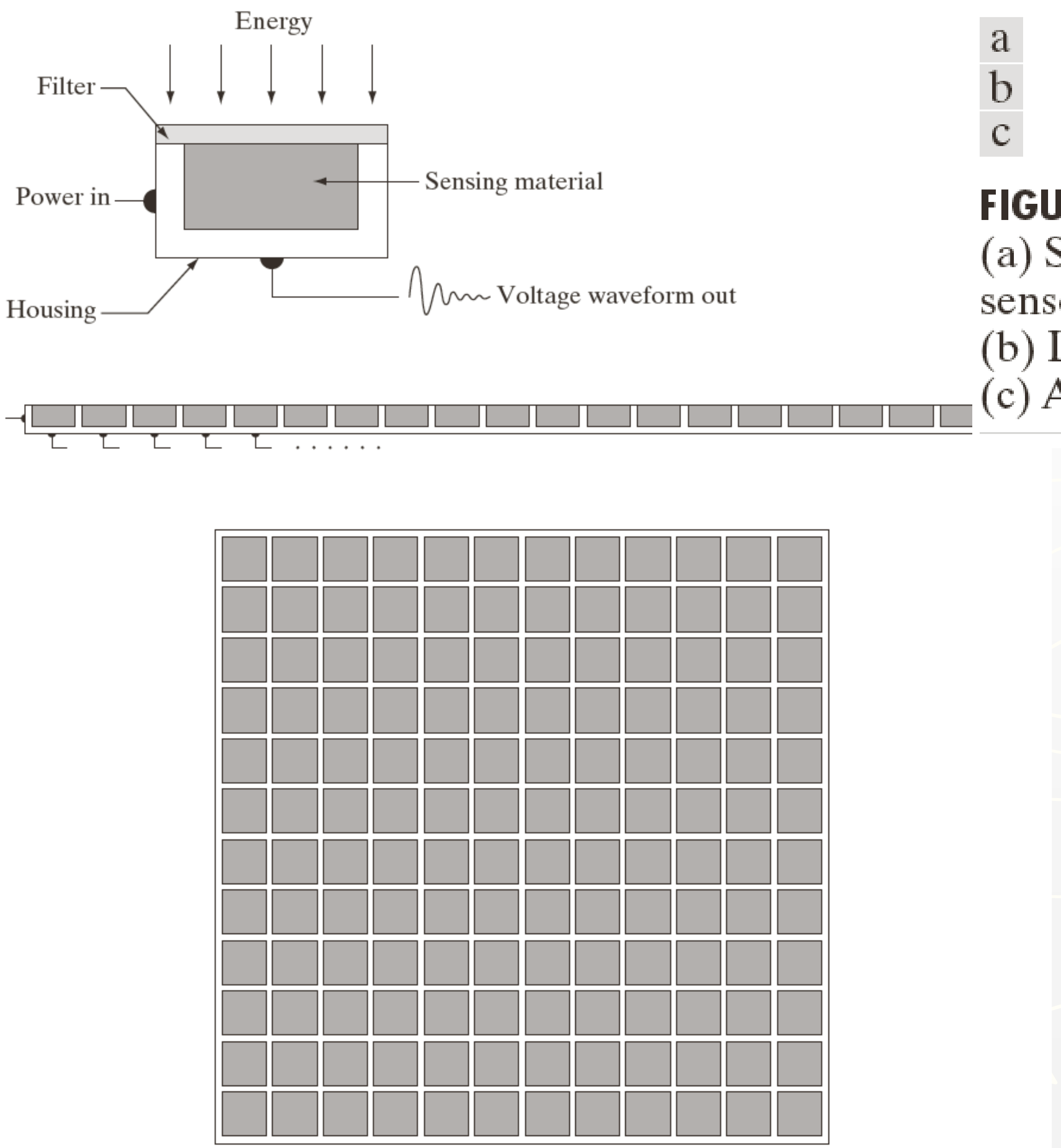
  The quality of a chromatic light source:

  Radiance: total amount of energy

  Luminance (lm): the amount of energy an observer perceives from a light source

  Brightness: a subjective descriptor of light perception that is impossible to measure. It embodies the achromatic notion of intensity and one of the key factors in describing color sensation.

Image Acquisition



a
b
c

FIGURE 2.12
(a) Single imaging sensor.
(b) Line sensor.
(c) Array sensor.

Transform illumination energy into digital images

Image Acquisition Using a Single Sensor

FIGURE 2.13
Combining a single sensor with motion to generate a 2-D image.

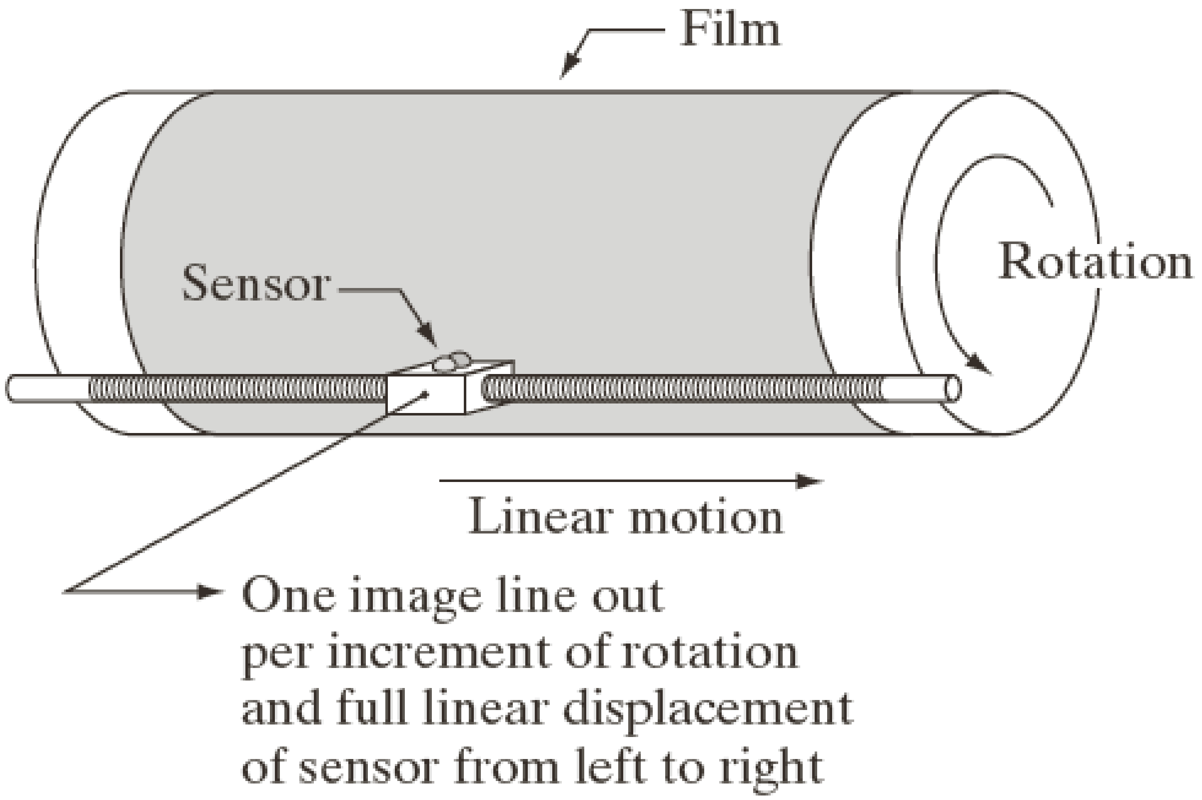
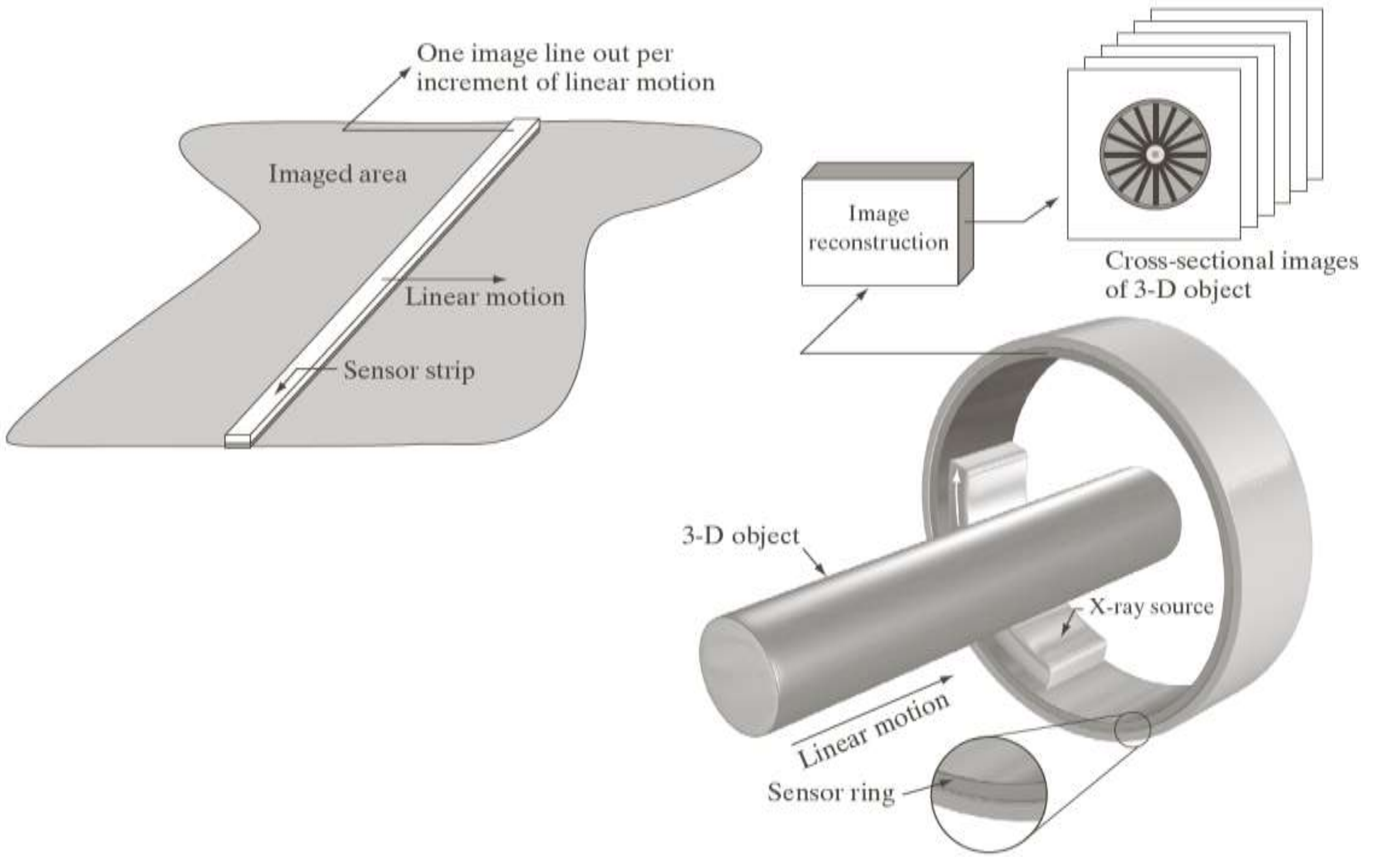


Image Acquisition Using Sensor Strips



a b

FIGURE 2.14 (a) Image acquisition using a linear sensor strip. (b) Image acquisition using a circular sensor strip.

Image Acquisition Process

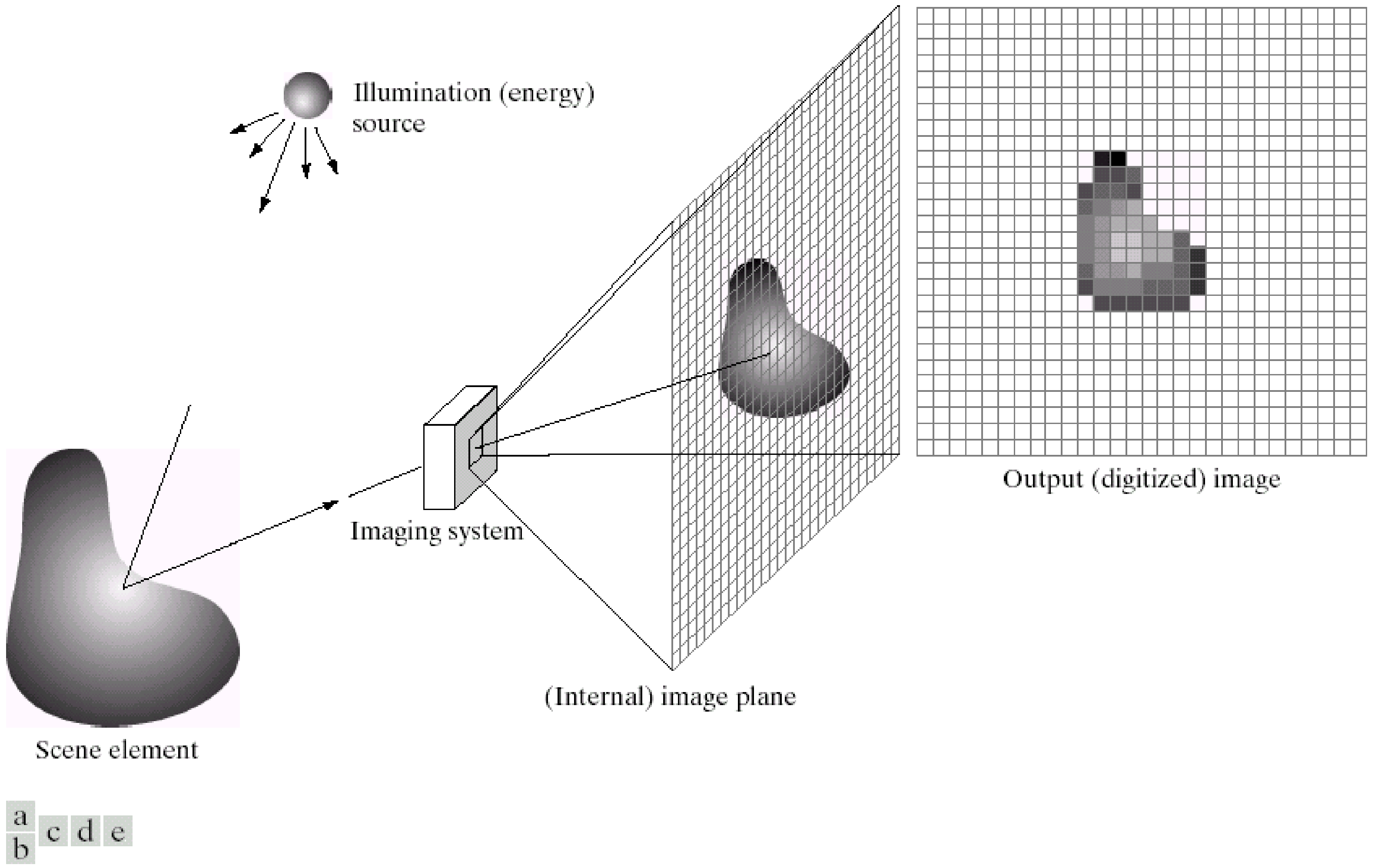


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

A Simple Image Formation Model

$$f(x, y) = i(x, y) * r(x, y)$$

$f(x, y)$: intensity at the point (x, y)

$i(x, y)$: illumination at the point (x, y)

(the amount of source illumination incident on the scene)

$r(x, y)$: reflectance/transmissivity at the point (x, y)

(the amount of illumination reflected/transmitted by the object)

where $0 < i(x, y) < \infty$ and $0 < r(x, y) < 1$

Some Typical Ranges of illumination

- Illumination

  Lumen — A unit of light flow or luminous flux

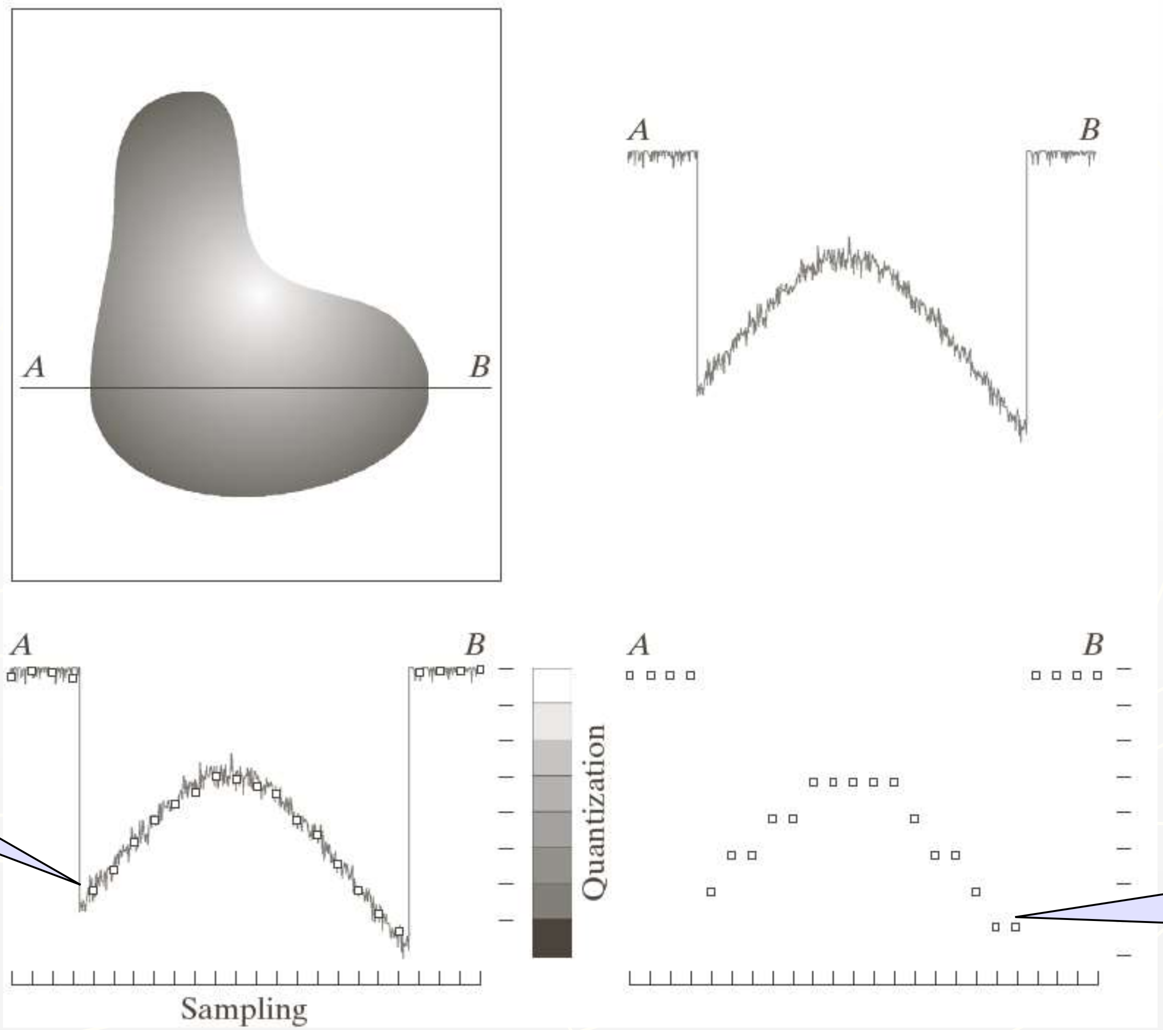
  Lumen per square meter (lm/m^2) — The metric unit of measure for illuminance of a surface

  - On a clear day, the sun may produce in excess of 90,000 lm/m^2 of illumination on the surface of the Earth
  - On a cloudy day, the sun may produce less than 10,000 lm/m^2 of illumination on the surface of the Earth
  - On a clear evening, the moon yields about 0.1 lm/m^2 of illumination
  - The typical illumination level in a commercial office is about 1000 lm/m^2

Some Typical Ranges of Reflectance

- Reflectance
  - 0.01 for black velvet
  - 0.65 for stainless steel
  - 0.80 for flat-white wall paint
  - 0.90 for silver-plated metal
  - 0.93 for snow

Image Sampling and Quantization



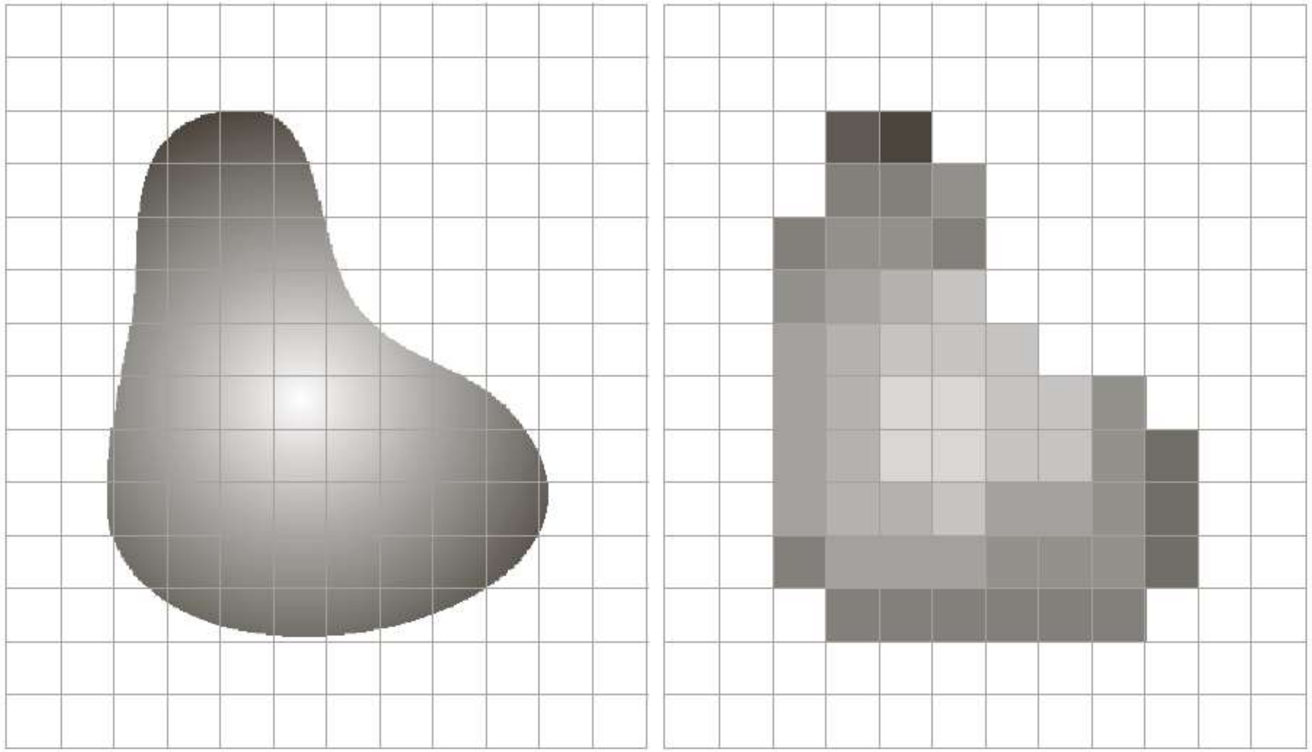
a b
c d

FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from *A* to *B* in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Digitizing the coordinate values

Digitizing the amplitude values

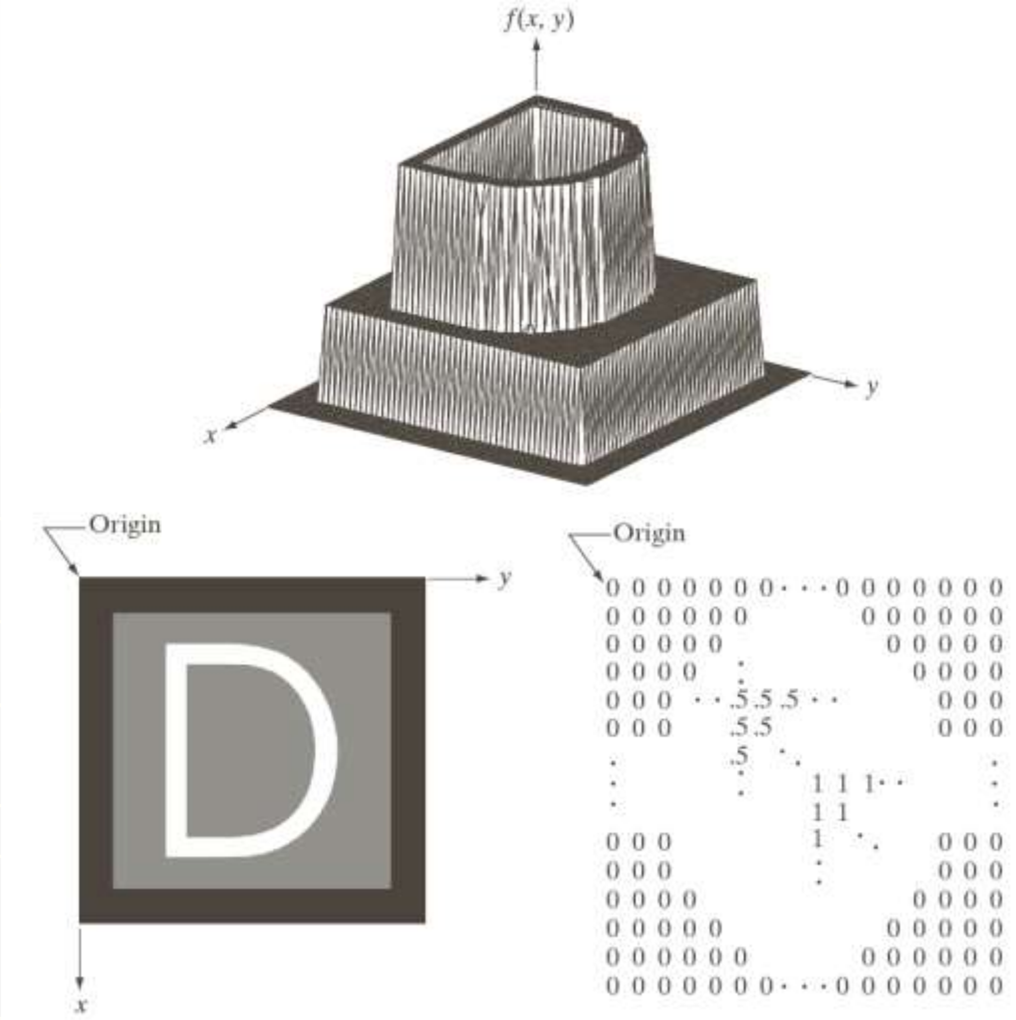
Image Sampling and Quantization



a b

FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

Representing Digital Images



a
b c

FIGURE 2.18
(a) Image plotted as a surface.
(b) Image displayed as a visual intensity array.
(c) Image shown as a 2-D numerical array (0, .5, and 1 represent black, gray, and white, respectively).

Representing Digital Images

- The representation of an M×N numerical array as

$$f(x,y)=\begin{bmatrix} f(0,0) & f(0,1) & \dots & f(0,N-1) \\ f(1,0) & f(1,1) & \dots & f(1,N-1) \\ \dots & \dots & \dots & \dots \\ f(M-1,0) & f(M-1,1) & \dots & f(M-1,N-1) \end{bmatrix}$$

Representing Digital Images

- The representation of an M×N numerical array as

$$A = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \cdots & \cdots & \cdots & \cdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}$$

Representing Digital Images

- The representation of an M×N numerical array in MATLAB

$$f(x,y)=\begin{bmatrix} f(1,1) & f(1,2) & \dots & f(1,N) \\ f(2,1) & f(2,2) & \dots & f(2,N) \\ \dots & \dots & \dots & \dots \\ f(M,1) & f(M,2) & \dots & f(M,N) \end{bmatrix}$$

Representing Digital Images

- Discrete intensity interval [0, L-1], $L=2^k$
- The number b of bits required to store a $M \times N$ digitized image

$$b = M \times N \times k$$

Representing Digital Images

TABLE 2.1
Number of storage bits for various values of N and k.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Digital Image Processing

Lecture 6
Introduction & Fundamentals

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Geometric Spatial Transformations

- Geometric transformation (rubber-sheet transformation)

  — A spatial transformation of coordinates

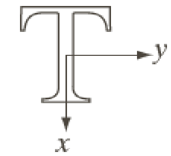
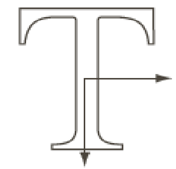
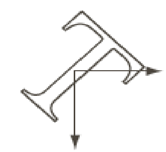
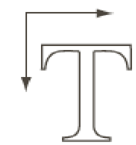
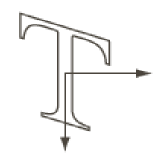
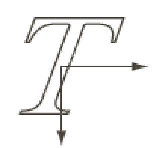
$$(x, y) = T\{(v, w)\}$$

  — intensity interpolation that assigns intensity values to the spatially transformed pixels.

- Affine transform

$$\begin{bmatrix} x & y & 1 \end{bmatrix} = \begin{bmatrix} v & w & 1 \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$$

TABLE 2.2
Affine transformations based on Eq. (2.6.–23).

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cos\theta + w\sin\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

Intensity Assignment

- Forward Mapping

$$(x, y) = T\{(v, w)\}$$

It's possible that two or more pixels can be transformed to the same location in the output image.

- Inverse Mapping

$$(v, w) = T^{-1}\{(x, y)\}$$

The nearest input pixels to determine the intensity of the output pixel value.

Inverse mappings are more efficient to implement than forward mappings.

Example: Image Rotation and Intensity Interpolation

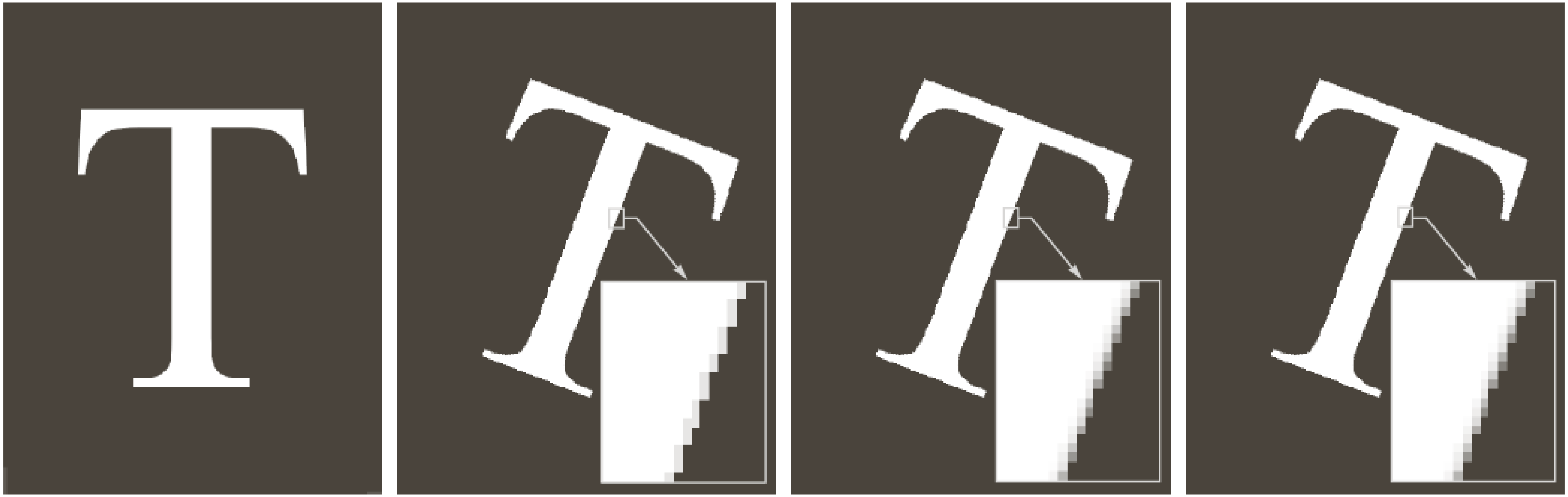


FIGURE 2.36 (a) A 300 dpi image of the letter T. (b) Image rotated 21° clockwise using nearest neighbor interpolation to assign intensity values to the spatially transformed pixels. (c) Image rotated 21° using bilinear interpolation. (d) Image rotated 21° using bicubic interpolation. The enlarged sections show edge detail for the three interpolation approaches.

Image Registration

- Input and output images are available but the transformation function is unknown.

  Goal: estimate the transformation function and use it to register the two images.

- One of the principal approaches for image registration is to use ***tie points*** (also called ***control points***)
  - The corresponding points are known precisely in the input and output (**reference**) images.

Image Registration

- A simple model based on bilinear approximation:

$$\begin{cases} x = c_1 v + c_2 w + c_3 vw + c_4 \\ y = c_5 v + c_6 w + c_7 vw + c_8 \end{cases}$$

Where (v, w) and (x, y) are the coordinates of tie points in the input and reference images.

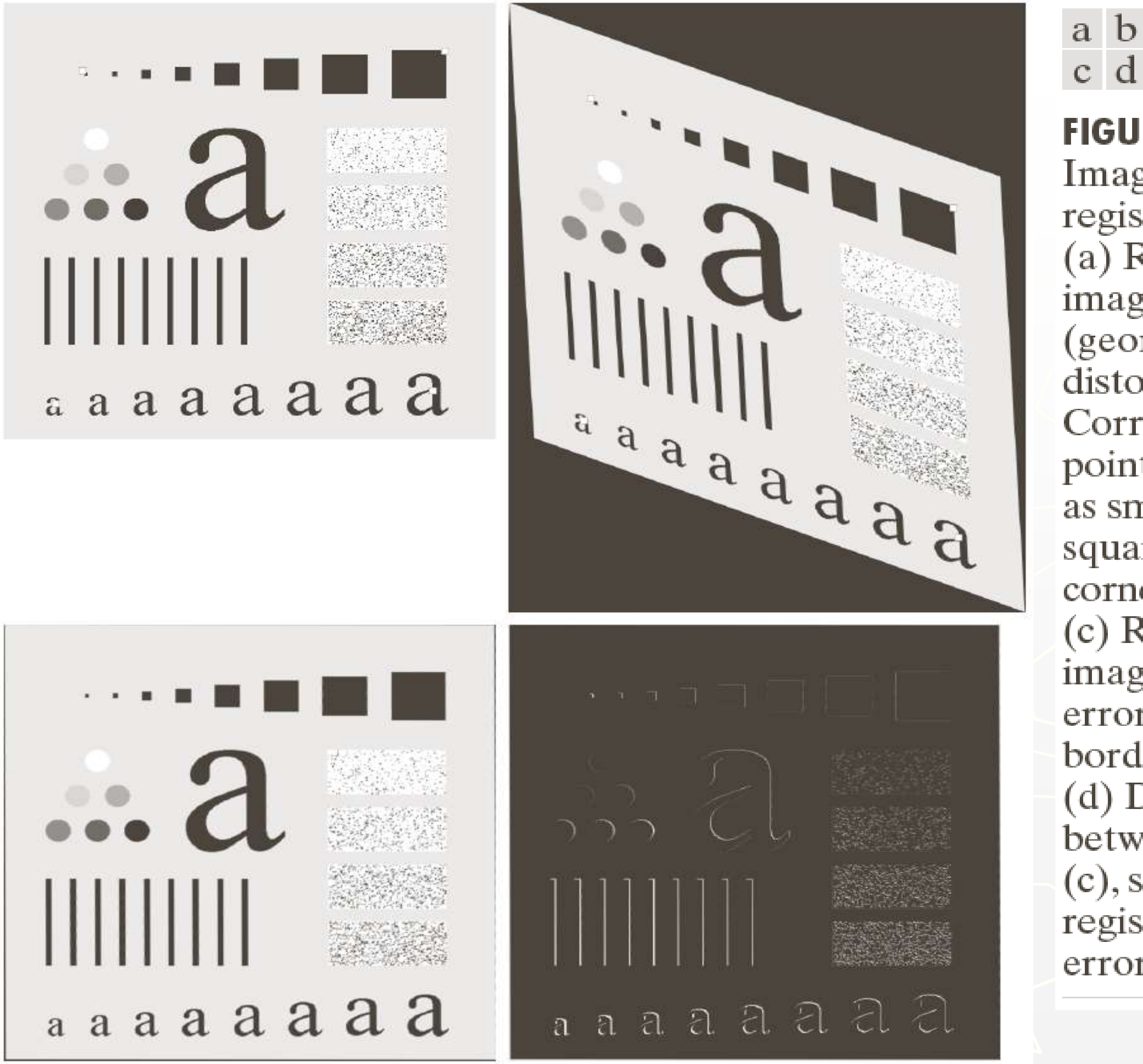


FIGURE 2.37 Image registration. (a) Reference image. (b) Input (geometrically distorted image). Corresponding tie points are shown as small white squares near the corners. (c) Registered image (note the errors in the borders). (d) Difference between (a) and (c), showing more registration errors.

Image Transform

- A particularly important class of 2-D linear transforms, denoted T(u, v)

$$T(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) r(x,y,u,v)$$

where $f(x,y)$ is the input image,

$r(x,y,u,v)$ is the *forward transformation* ker*nel*,

variables u and v are the transform variables,

u = 0, 1, 2, ..., M-1 and v = 0, 1, ..., N-1.

Image Transform

- Given T(u, v), the original image f(x, y) can be recoverd using the inverse tranformation of T(u, v).

$$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} T(u,v)s(x,y,u,v)$$

where $s(x,y,u,v)$ is the *inverse transformation kernel*,

x = 0, 1, 2, ..., M-1 and y = 0, 1, ..., N-1.

Image Transform

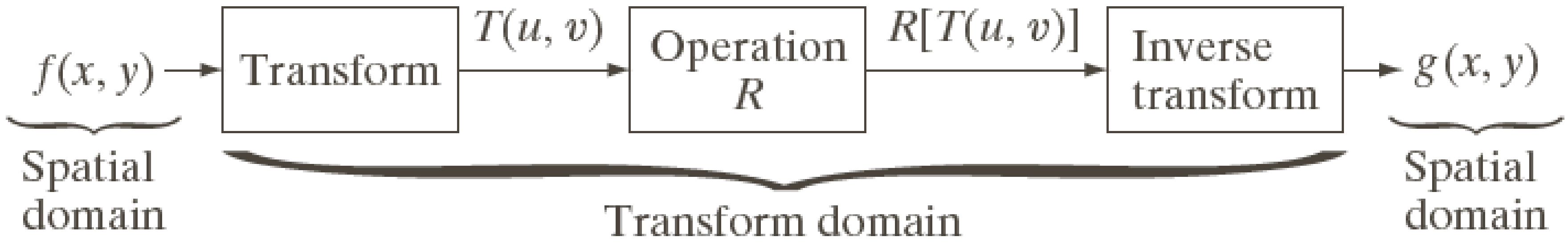


FIGURE 2.39 General approach for operating in the linear transform domain.

Example: Image Denoising by Using DCT Transform

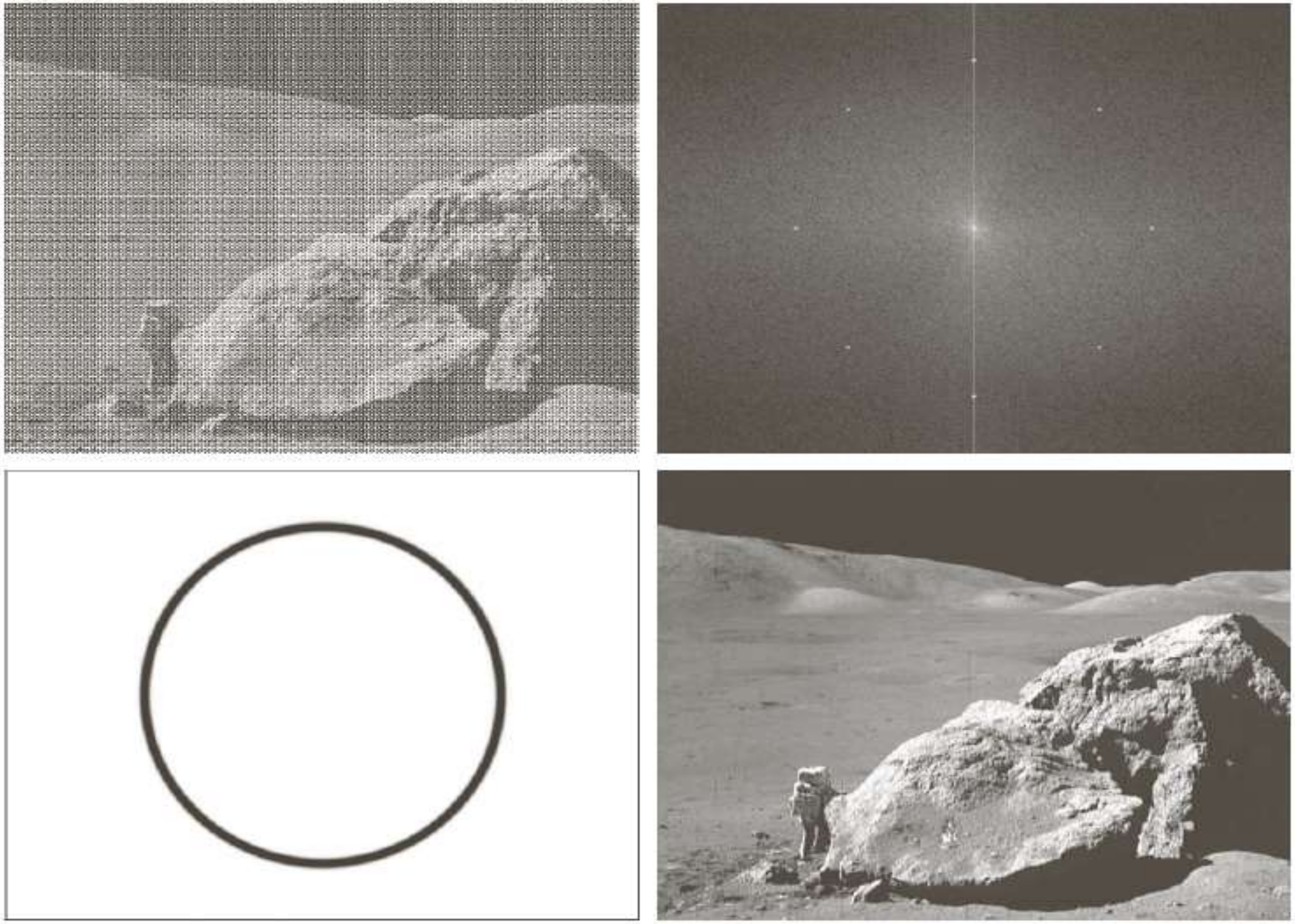


FIGURE 2.40
(a) Image corrupted by sinusoidal interference. (b) Magnitude of the Fourier transform showing the bursts of energy responsible for the interference. (c) Mask used to eliminate the energy bursts. (d) Result of computing the inverse of the modified Fourier transform. (Original image courtesy of NASA.)

Forward Transform Kernel

$$T(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)r(x,y,u,v)$$

The kernel $r(x,y,u,v)$ is said to be SEPERABLE if

$$r(x,y,u,v)=r_1(x,u)r_2(y,v)$$

In addition, the kernel is said to be SYMMETRIC if

$r_1(x,u)$ is functionally equal to $r_2(y,v),$ so that

$$r(x,y,u,v)=r_1(x,u)r_1(y,u)$$

The Kernels for 2-D Fourier Transform

The *forward* kernel

$$r(x, y, u, v) = e^{-j2\pi(ux/M + vy/N)}$$

Where $j = \sqrt{-1}$

The *inverse* kernel

$$s(x, y, u, v) = \frac{1}{MN} e^{j2\pi(ux/M + vy/N)}$$

2-D Fourier Transform

$$T(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

$$f(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} T(u,v)e^{j2\pi(ux/M+vy/N)}$$

Probabilistic Methods

Let $z_i,\ i = 0,\ 1,\ 2,\ ...,\ L-1$, denote the values of all possible intensities in an $M \times N$ digital image. The probability, $p(z_k)$, of intensity level z_k occurring in a given image is estimated as

$$p(z_k) = \frac{n_k}{MN},$$

where n_k is the number of times that intensity z_k occurs in the image.

$$\sum_{k=0}^{L-1} p(z_k) = 1$$

The mean (average) intensity is given by

$$m = \sum_{k=0}^{L-1} z_k\, p(z_k)$$

Probabilistic Methods

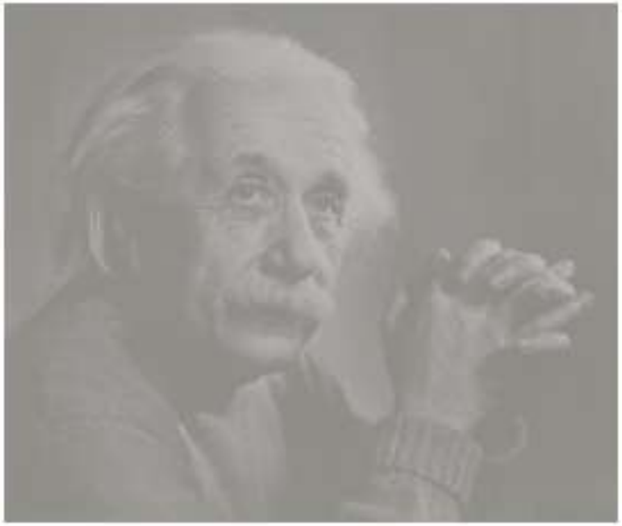
The variance of the intensities is given by

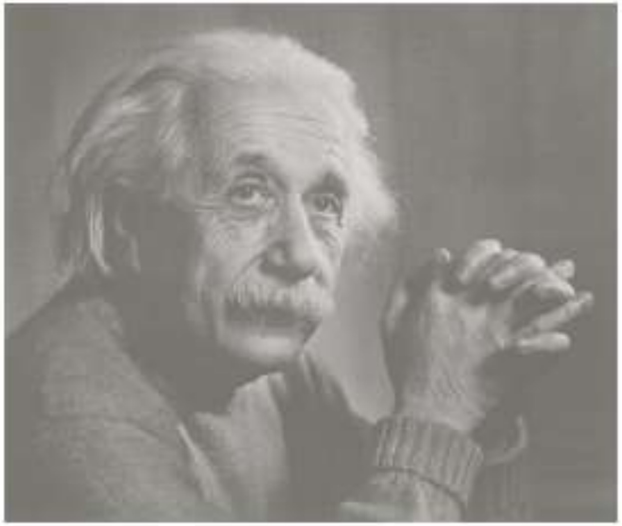
$$\sigma^2 = \sum_{k=0}^{L-1} (z_k - m)^2 p(z_k)$$

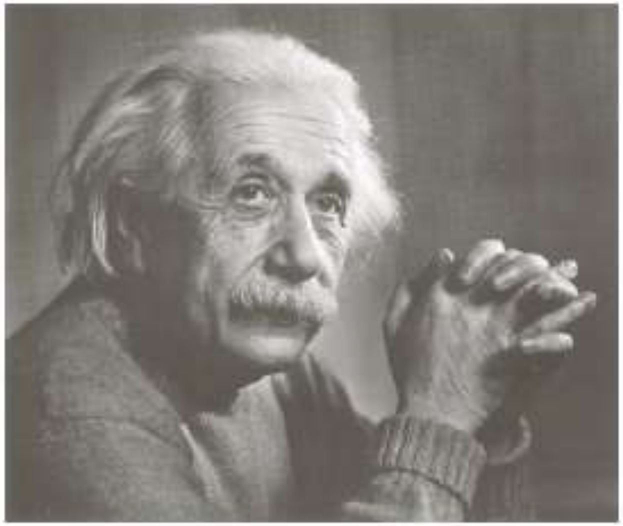
The n^{th} moment of the intensity variable z is

$$u_n(z) = \sum_{k=0}^{L-1} (z_k - m)^n p(z_k)$$

Example: Comparison of Standard Deviation Values







$\sigma = 14.3$ $\sigma = 31.6$ $\sigma = 49.2$

Digital Image Processing

Lecture 14
Filtering in the Frequency Domain

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Image Sharpening Using Frequency Domain Filters

A highpass filter is obtained from a given lowpass filter using

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

A 2-D ideal highpass filter (IHPL) is defined as

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

Image Sharpening Using Frequency Domain Filters

A 2-D Butterworth highpass filter (BHPL) is defined as

$$H(u,v) = \frac{1}{1+\left[D_0 / D(u,v)\right]^{2n}}$$

A 2-D Gaussian highpass filter (GHPL) is defined as

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

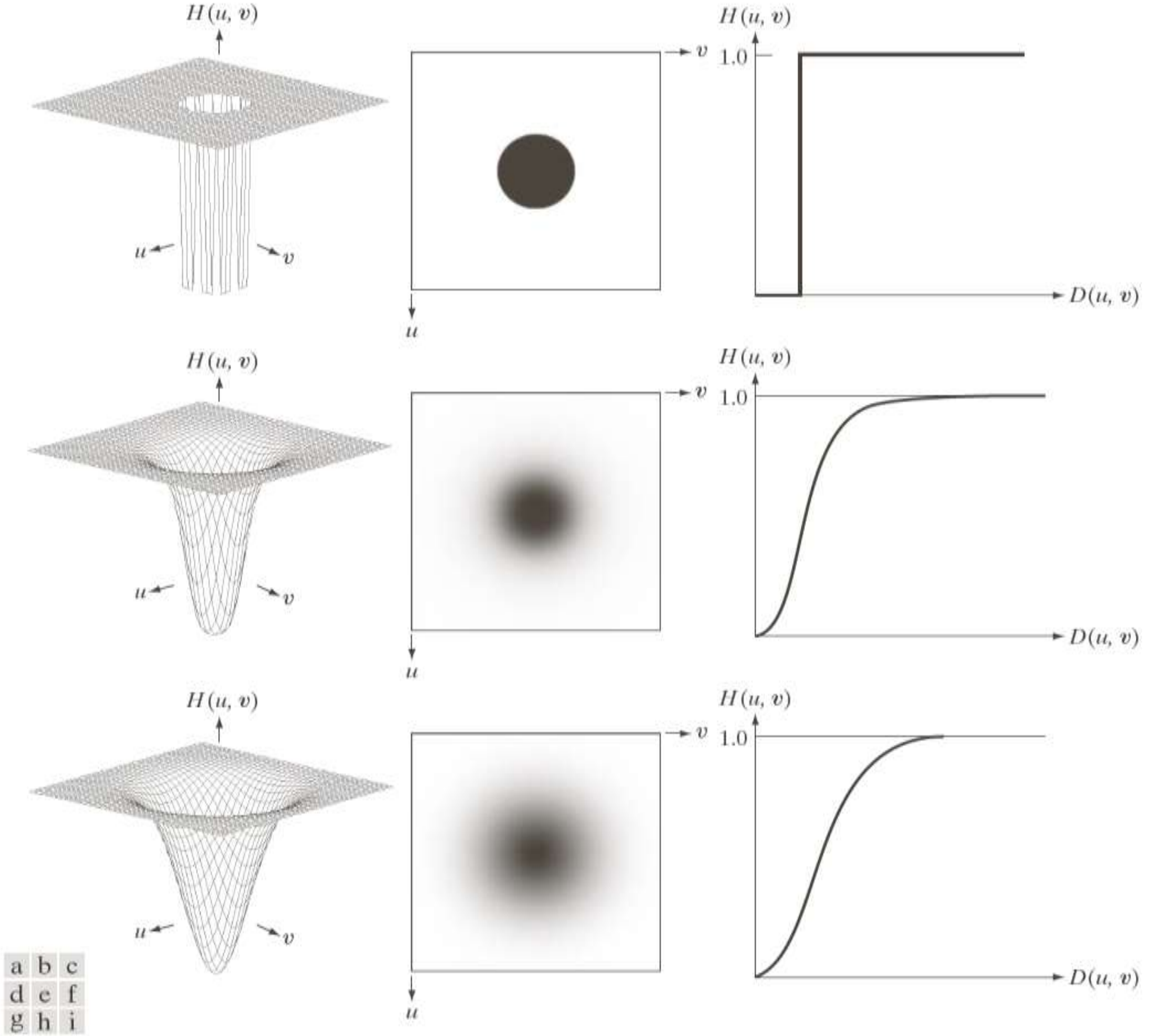
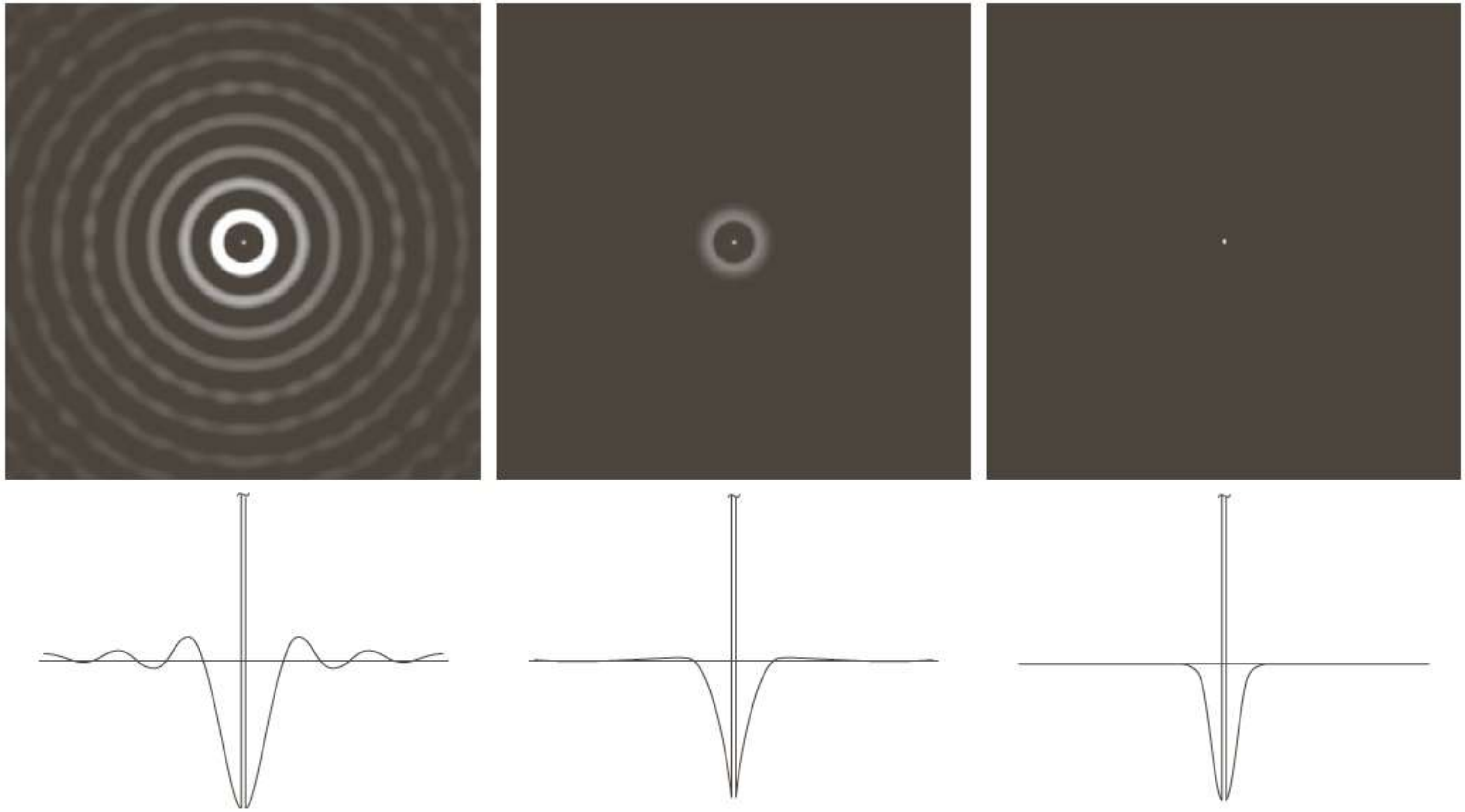


FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

The Spatial Representation of Highpass Filters



a b c

FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

Filtering Results by IHPF

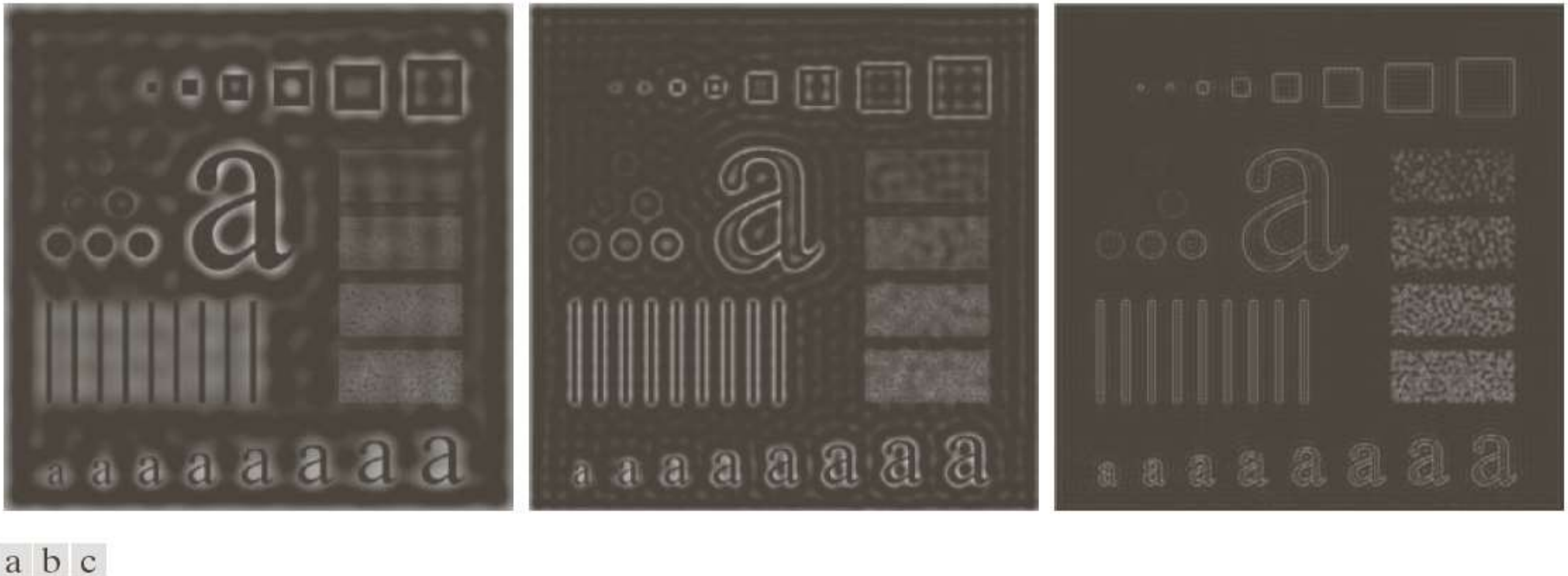


FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30, 60,$ and 160.

Filtering Results by BHPF

a b c

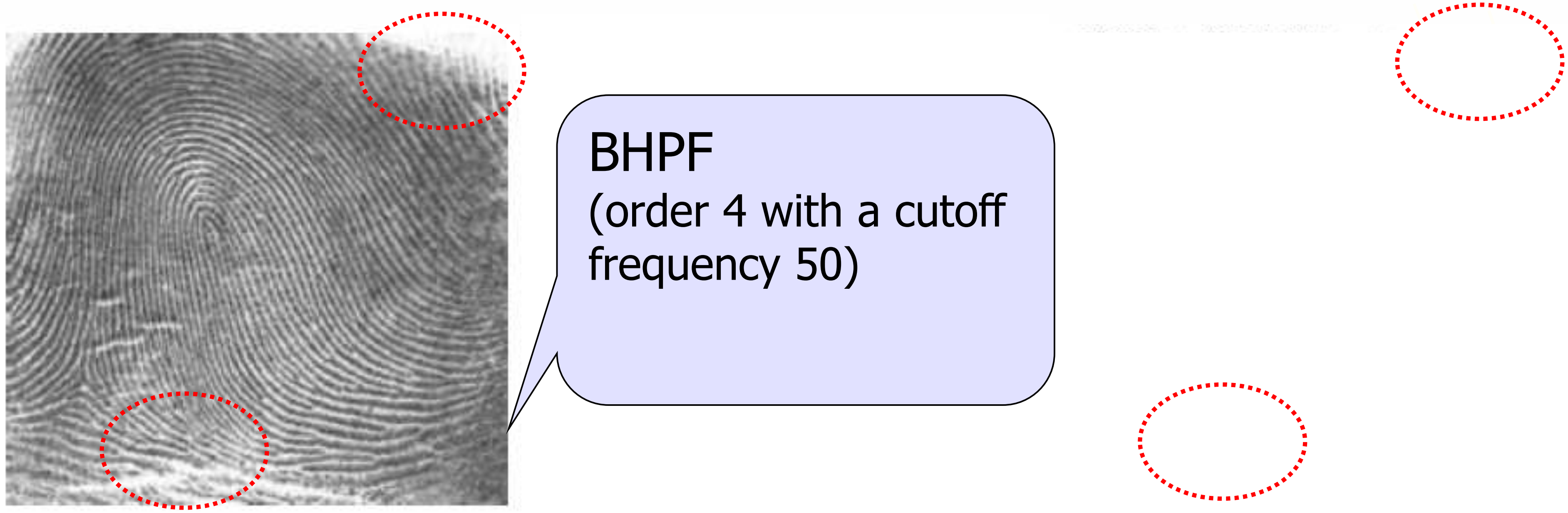
FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

Filtering Results by GHPF

a b c

FIGURE 4.56 Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.

Using Highpass Filtering and Threshold for Image Enhancement



a b c

FIGURE 4.57 (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

The Laplacian in the Frequency Domain

$$H(u,v) = -4\pi^2(u^2 + v^2)$$

$$H(u,v) = -4\pi^2\left[(u - P/2)^2 + (v - Q/2)^2)\right]$$
$$= -4\pi^2 D^2(u,v)$$

The Laplacian image

$$\nabla^2 f(x,y) = \Im^{-1}\left\{H(u,v)F(u,v)\right\}$$

Enhancement is obtained

$$g(x,y) = f(x,y) + c\nabla^2 f(x,y) \qquad c = \text{-}1$$

The Laplacian in the Frequency Domain

The enhanced image

$$
\begin{aligned}
g(x,y) &= \Im^{-1}\left\{F(u,v) - H(u,v)F(u,v)\right\} \\
&= \Im^{-1}\left\{\left[1 - H(u,v)\right]F(u,v)\right\} \\
&= \Im^{-1}\left\{\left[1 + 4\pi^2 D^2(u,v)\right]F(u,v)\right\}
\end{aligned}
$$

The Laplacian in the Frequency Domain

a b

FIGURE 4.58
(a) Original, blurry image.
(b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).

Unsharp Masking, Highboost Filtering and High-Frequency-Emphasis Fitering

$$g_{mask}(x,y) = f(x,y) - f_{LP}(x,y)$$

$$f_{LP}(x,y) = \mathfrak{I}^{-1}\left[H_{LP}(u,v)F(u,v)\right]$$

Unsharp masking and highboost filtering

$$g(x,y) = f(x,y) + k * g_{mask}(x,y)$$

$$g(x,y) = \mathfrak{I}^{-1}\left\{\left[1 + k * \left[1 - H_{LP}(u,v)\right]\right]F(u,v)\right\}$$

$$= \mathfrak{I}^{-1}\left\{\left[1 + k * H_{HP}(u,v)\right]F(u,v)\right\}$$

Unsharp Masking, Highboost Filtering and High-Frequency-Emphasis Fitering

$$g(x,y) = \Im^{-1}\left\{\left[k_1 + k_2 * H_{HP}(u,v)\right]F(u,v)\right\}$$

$$k_1 \geq 0 \text{ and } k_2 \geq 0$$

Gaussian Filter
D_0=40

High-Frequency-Emphasis Filtering
Gaussian Filter
K1=0.5, k2=0.75

a	b
c	d

FIGURE 4.59 (a) A chest X-ray image. (b) Result of highpass filtering with a Gaussian filter. (c) Result of high-frequency-emphasis filtering using the same filter. (d) Result of performing histogram equalization on (c). (Original image courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

Homomorphic Filtering

$$f(x,y) = i(x,y)r(x,y)$$

$$\Im\left[f(x,y)\right] = \Im\left[i(x,y)\right]\Im\left[r(x,y)\right] \; ?$$

$$z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$$

$$\Im\left\{z(x,y)\right\} = \Im\left\{\ln f(x,y)\right\} = \Im\left\{\ln i(x,y)\right\} + \Im\left\{\ln r(x,y)\right\}$$

$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

Homomorphic Filtering

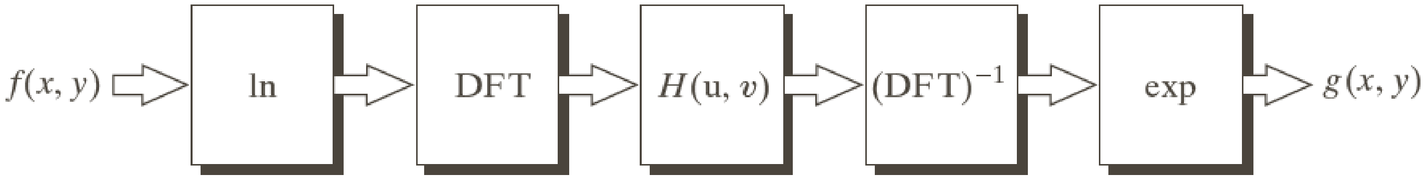
$$S(u,v) = H(u,v)Z(u,v)$$
$$= H(u,v)F_i(u,v) + H(u,v)F_r(u,v)$$

$$s(x,y) = \Im^{-1}\left\{S(u,v)\right\}$$
$$= \Im^{-1}\left\{H(u,v)F_i(u,v) + H(u,v)F_r(u,v)\right\}$$
$$= \Im^{-1}\left\{H(u,v)F_i(u,v)\right\} + \Im^{-1}\left\{H(u,v)F_r(u,v)\right\}$$
$$= i'(x,y) + r'(x,y)$$

$$g(x,y) = e^{s(x,y)} = e^{i'(x,y)}e^{r'(x,y)} = i_0(x,y)r_0(x,y)$$

Homomorphic Filtering

FIGURE 4.60
Summary of steps in homomorphic filtering.



The illumination component of an image generally is characterized by slow spatial variations, while the reflectance component tends to vary abruptly

These characteristics lead to associating the low frequencies of the Fourier transform of the logarithm of an image with illumination the high frequencies with reflectance.

Homomorphic Filtering

$$H(u,v)=(\gamma_H-\gamma_L)\left[1-e^{-c\left[D^2(u,v)/D_0^2\right]}\right]+\gamma_L$$

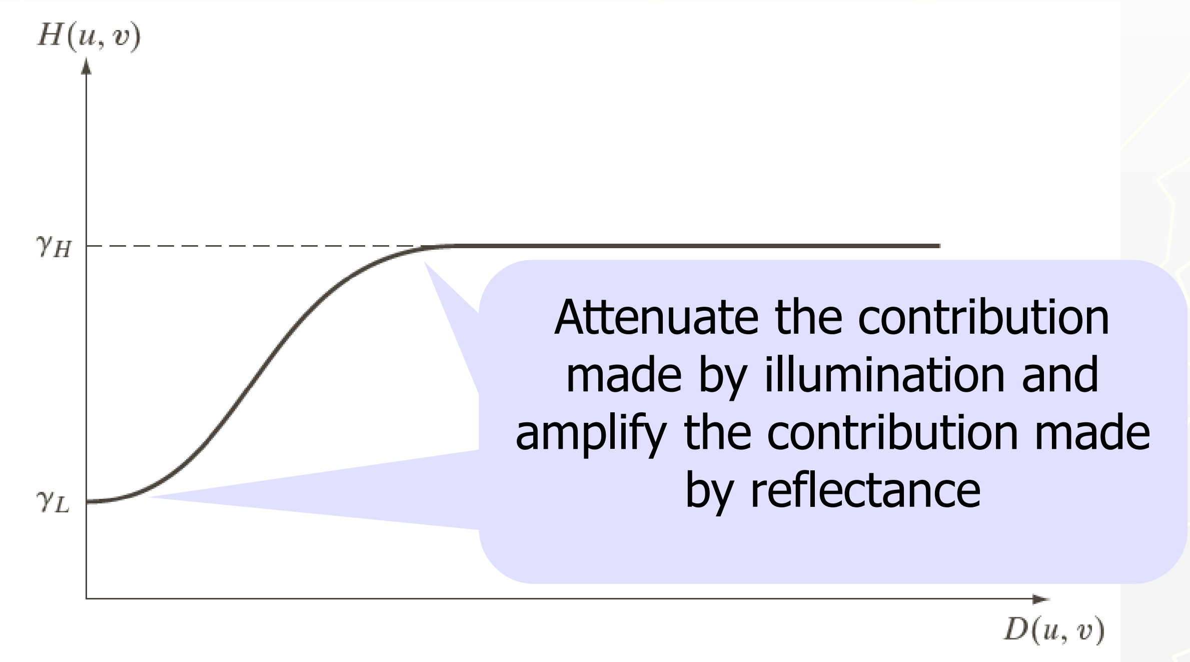


FIGURE 4.61 Radial cross section of a circularly symmetric homomorphic filter function. The vertical axis is at the center of the frequency rectangle and $D(u, v)$ is the distance from the center.

$\gamma_L = 0.25$

$\gamma_H = 2$

$c = 1$

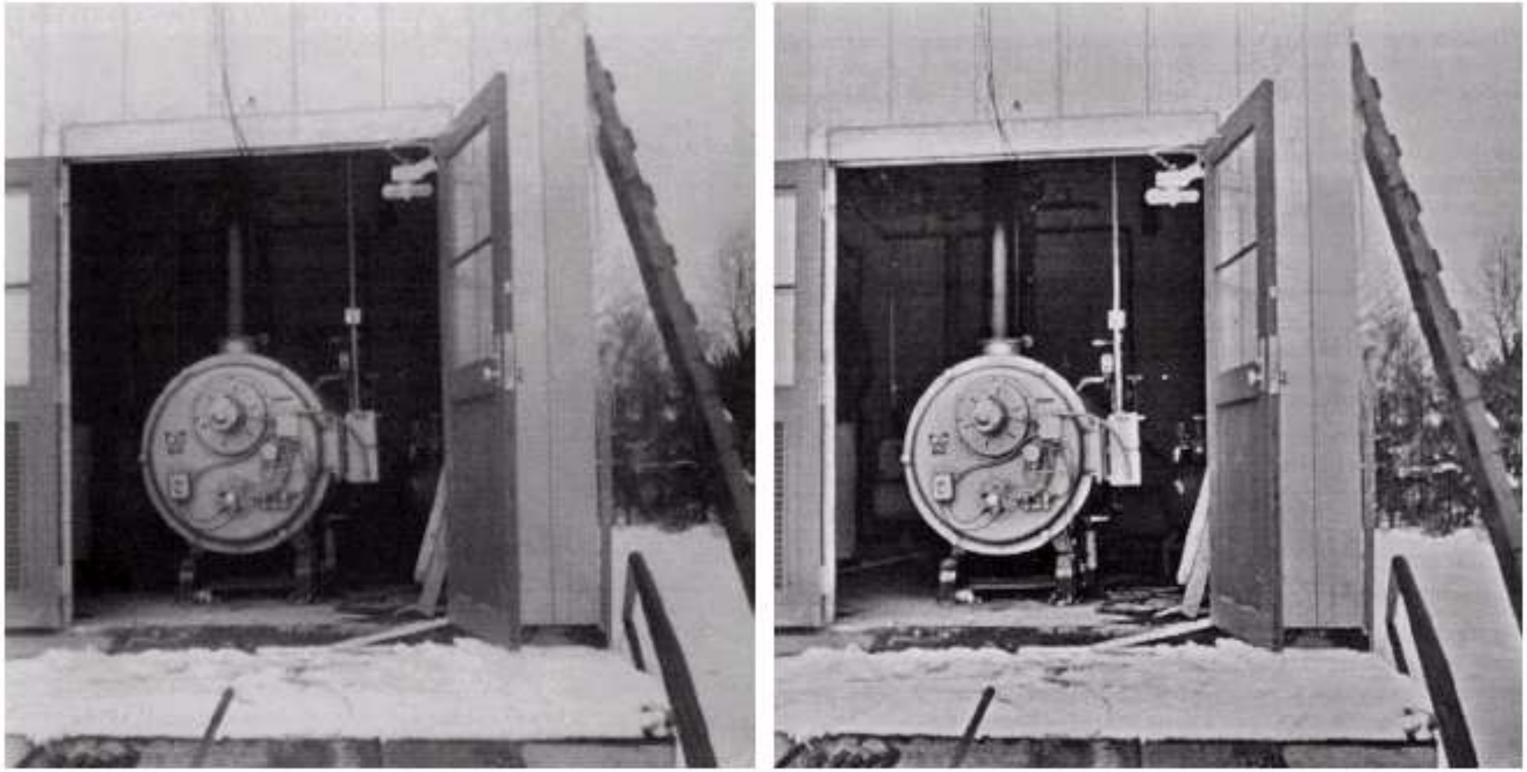
$D_0 = 80$

E 4.62
ll body PET
b) Image
ced using
morphic
ng. (Original
courtesy of
ichael
sey, CTI
Systems.)

Homomorphic Filtering

a b

FIGURE
(a) Original image. (b) Image processed by homomorphic filtering (note details inside shelter). (Stockham.)



Selective Filtering

Non-Selective Filters:
operate over the entire frequency rectangle

Selective Filters
operate over some part, not entire frequency rectangle
- **bandreject or bandpass**: process specific bands
- **notch filters**: process small regions of the frequency rectangle

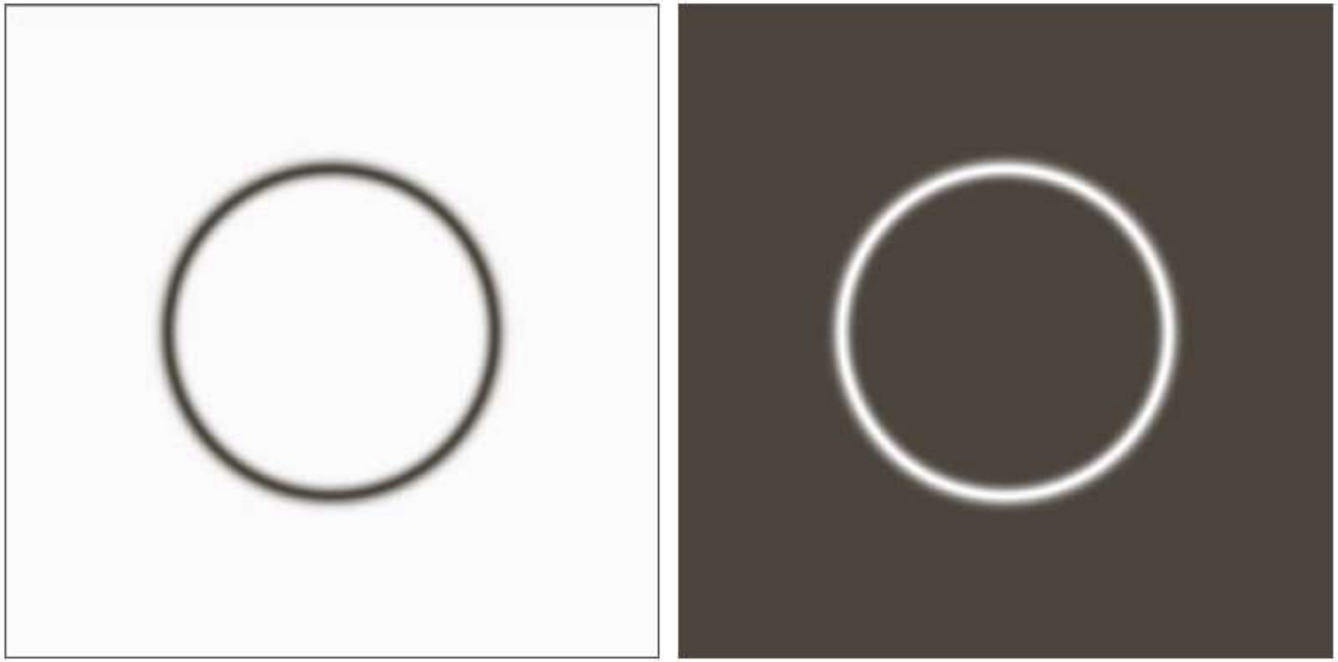
Selective Filtering: Bandreject and Bandpass Filters

TABLE 4.6
Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

Selective Filtering: Bandreject and Bandpass Filters



a b

FIGURE 4.63 (a) Bandreject Gaussian filter. (b) Corresponding bandpass filter. The thin black border in (a) was added for clarity; it is not part of the data.

Selective Filtering: Notch Filters

Zero-phase-shift filters must be symmetric about the origin. A notch with center at (u_0, v_0) must have a corresponding notch at location $(-u_0,-v_0)$.

Notch reject filters are constructed as products of highpass filters whose centers have been translated to the centers of the notches.

$$H_{NR}(u,v) = \prod_{k=1}^{Q} H_k(u,v) H_{-k}(u,v)$$

where $H_k(u,v)$ and $H_{-k}(u,v)$ are highpass filters whose centers are at (u_k, v_k) and $(-u_k, -v_k)$, respectively.

Selective Filtering: Notch Filters

$$H_{NR}(u,v)=\prod_{k=1}^{Q}H_k(u,v)H_{-k}(u,v)$$

where $H_k(u,v)$ and $H_{-k}(u,v)$ are highpass filters whose centers are at (u_k, v_k) and $(-u_k, -v_k)$, respectively.

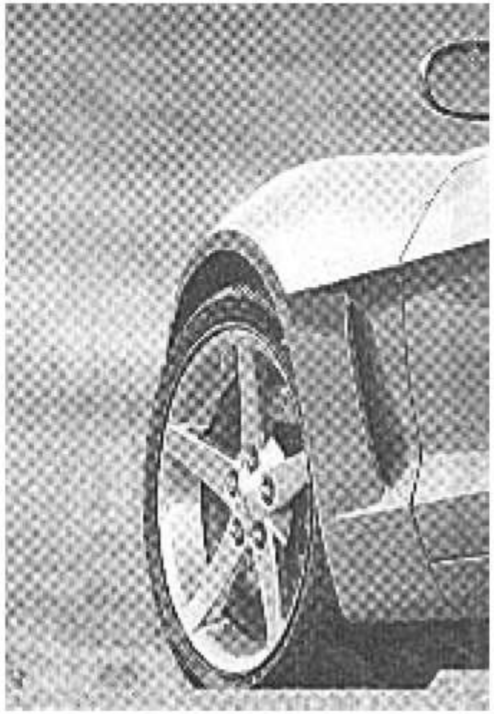
A Butterworth notch reject filter of order n

$$H_{NR}(u,v)=\prod_{k=1}^{3}\left[\frac{1}{1+\left[D_{0k}/D_k(u,v)\right]^{2n}}\right]\left[\frac{1}{1+\left[D_{0k}/D_{-k}(u,v)\right]^{2n}}\right]$$

$$D_k(u,v)=\left[(u-M/2-u_k)^2+(v-N/2-v_k)^2\right]^{1/2}$$

$$D_{-k}(u,v)=\left[(u-M/2+u_k)^2+(v-N/2+v_k)^2\right]^{1/2}$$

Examples: Notch Filters (1)

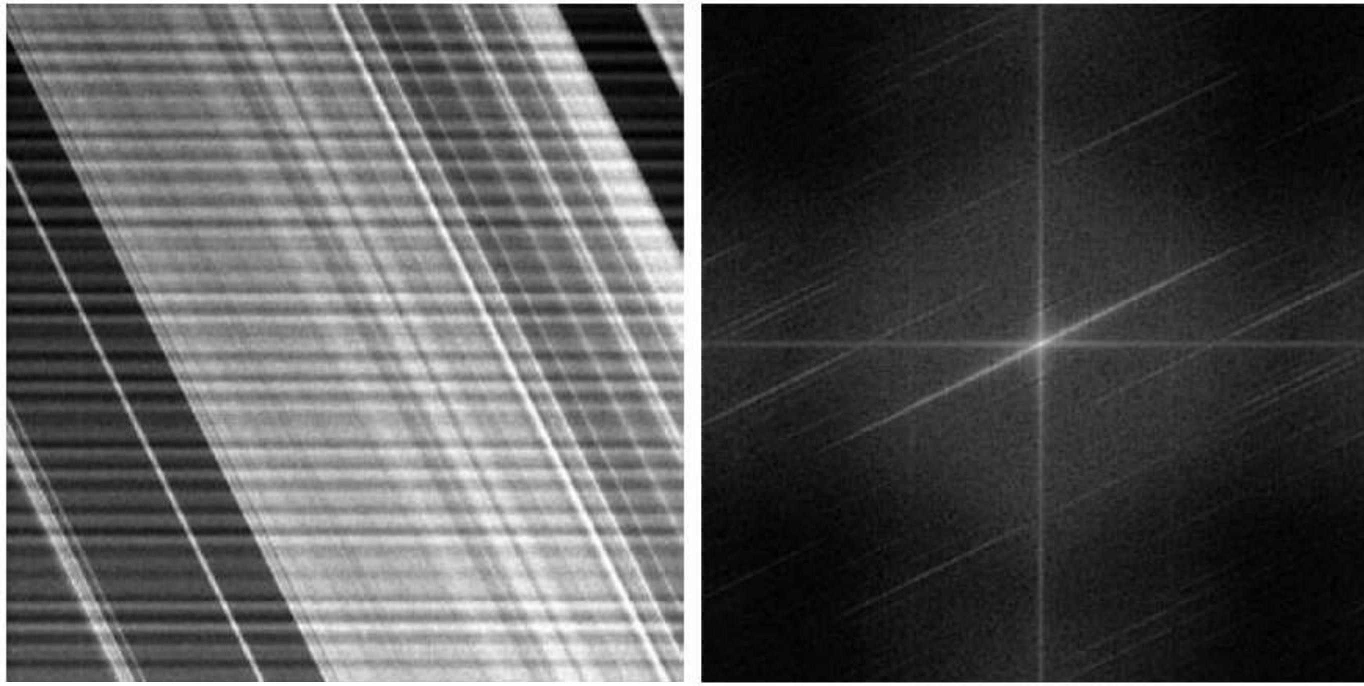


a b
c d

FIGURE 4.64 (a) Sampled newspaper image showing a moiré pattern. (b) Spectrum. (c) Butterworth notch reject filter multiplied by the Fourier transform. (d) Filtered image.

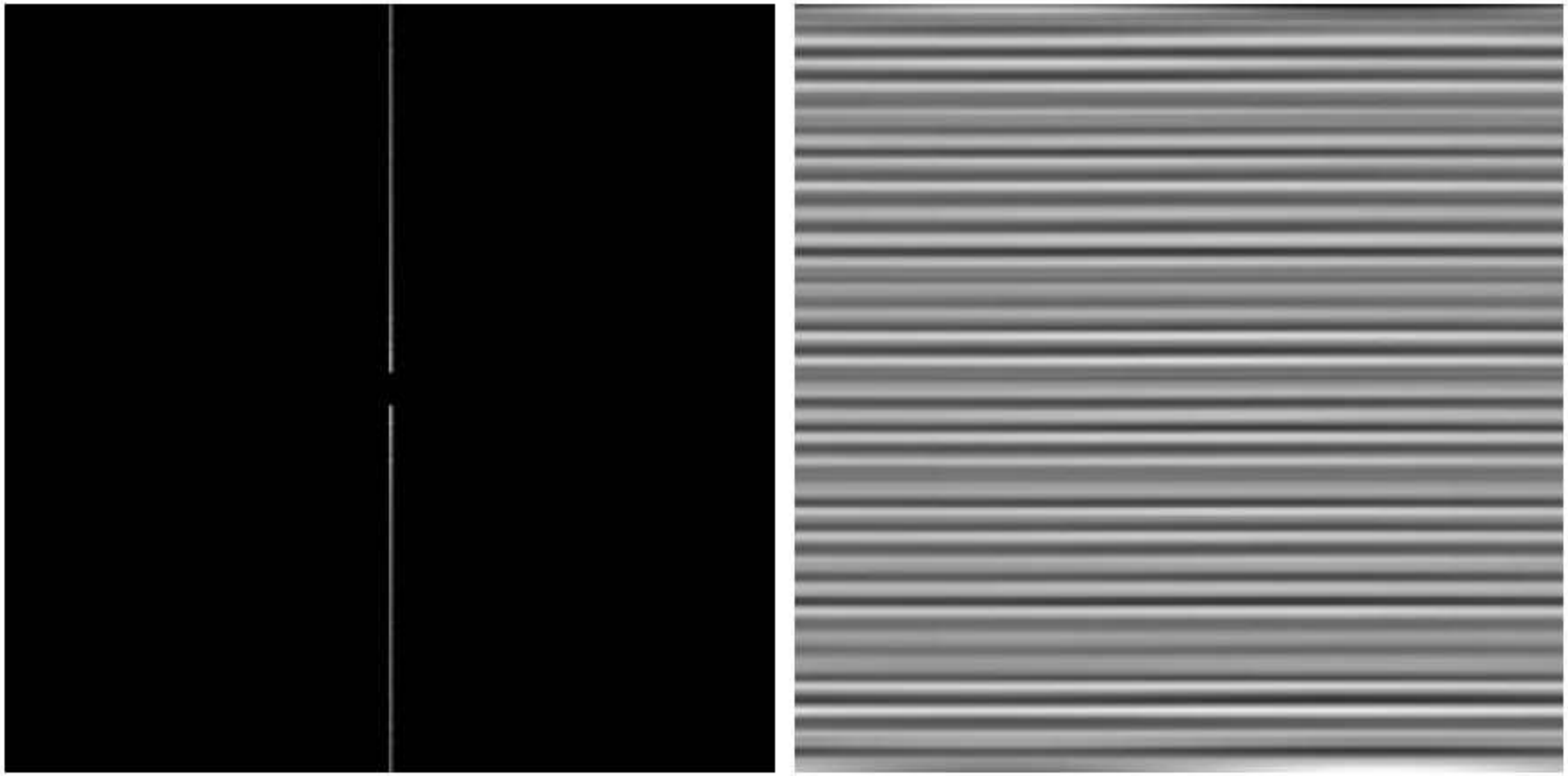
A Butterworth notch reject filter $D_0 = 3$ and $n = 4$ for all notch pairs

Examples: Notch Filters (2)



a b
c d

FIGURE 4.65
(a) 674 × 674 image of the Saturn rings showing nearly periodic interference. (b) Spectrum: The bursts of energy in the vertical axis near the origin correspond to the interference pattern. (c) A vertical notch reject filter. (d) Result of filtering. The thin black border in (c) was added for clarity; it is not part of the data. (Original image courtesy of Dr. Robert A. West, NASA/JPL.)



a b

FIGURE 4.66
(a) Result (spectrum) of applying a notch pass filter to the DFT of Fig. 4.65(a).
(b) Spatial pattern obtained by computing the IDFT of (a).